AP Calculus BC

Extreme Values

1) $f(x) = 3x^{2/3} - 2x$

a) D: $(-\infty, \infty)$

b) $f'(x) = 2x^{-1/3} - 2$

D: $(-\infty, 0), (0, \infty)$

c) $\frac{2}{x^{1/3}} - 2 = 0$

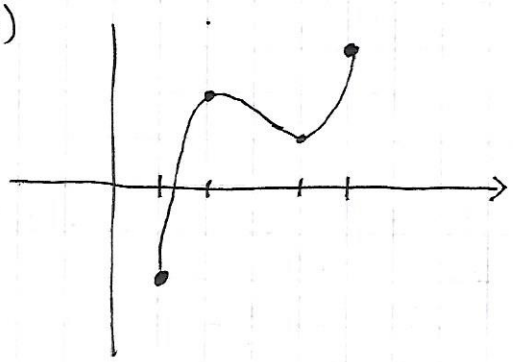
$x = 1$

x	f(x)
-1	5
0	0
1	1
2	$3\sqrt[3]{4} - 4$

abs. max: $(-1, 5)$

abs. min: $(0, 0)$

d)



2) a)

abs max: 2

abs min: 0

b)

abs max: $f(c)$

No abs min

c)

abs. max: $h(b)$

abs. min: $h(c_2)$

3) a) $x(t) = 3t^4 + 4t^3 - 6t^2$

$x'(t) = 12t^3 + 12t^2 - 12t = 0$

$= 12t(t^2 + t - 1)$

$t = 0 \quad t = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$

$t = \frac{-1 \pm \sqrt{5}}{2}$

b) $g(t) = 5t^{2/3} + t^{5/3}$

$g'(t) = \frac{10}{3}t^{-1/3} + \frac{5}{3}t^{2/3}$

$t = 0$

$\frac{10}{3t^{1/3}} + \frac{5t^{2/3}}{3} = 0$

$\frac{10}{3} + \frac{5t}{3} = 0$

$t = 2$

c) $f(x) = x \ln x \quad D: (0, \infty)$

$f'(x) = x \cdot \frac{1}{x} + \ln x$

$= 1 + \ln x$

$1 + \ln x = 0$

$\ln x = -1$

$x = \frac{1}{e}$

d) $g(\theta) = 4\theta - \tan\theta$

$g'(\theta) = 4 - \sec^2\theta$

$4 - \sec^2\theta = 0$

$4 = \sec^2\theta$

$\frac{1}{4} = \cos^2\theta$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4} + \frac{k\pi}{2}$

e) $f(x) = xe^{2x}$

$f'(x) = 2xe^{2x} + e^{2x} = 0$

$e^{2x}(2x+1) = 0$

$\boxed{x = -\frac{1}{2}}$

4) a) $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$

$f'(x) = 6x^2 - 6x - 12 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

x	$f(x)$	
-2	-3	
-1	8	← abs max
2	-19	← abs min
3	-8	

b) $f(x) = (x^2-1)^3$ on $[-1, 2]$

$f'(x) = 3(x^2-1)^2(2x) = 0$

$x = \pm 1 \quad x = 0$

x	$f(x)$	
-1	-8	← abs min
0	-1	
1	0	
2	27	← abs max

c) $f(x) = \sin(x + \frac{\pi}{4}) \quad [0, \frac{7\pi}{4}]$

$f'(x) = \cos(x + \frac{\pi}{4})$

$\cos(x + \frac{\pi}{4}) = 0$

$x + \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}, \cancel{\frac{5\pi}{2}}$

$x = \frac{\pi}{4}, \frac{5\pi}{4},$

x	$f(x)$	
0	$\sqrt{2}/2$	← abs. max
$\pi/4$	1	
$5\pi/4$	-1	← abs min
$7\pi/4$	0	

d) $k(x) = e^{-x^2}$

$k'(x) = -2x\,e^{-x^2} = 0$

$x = 0$

$k(0) = 1$ ← abs. max

$\lim_{x \to \infty} e^{-x^2} = 0 \quad \lim_{x \to -\infty} e^{-x^2} = 0$

5) $f(x) = \frac{5+2x}{x^2+14}$

$f'(x) = \frac{2(x^2+14) - 2x(5+2x)}{(x^2+14)^2} = 0$

$= 2x^2 + 28 - 10x - 4x^2 = 0$

$-2x^2 - 10x + 28 = 0$

$x^2 + 5x - 14 = 0$

$(x+7)(x-2) = 0$

x	$f(x)$	
-2	$1/18$	→
2	$1/2$	(D)
4	$13/30$	

6) $f(x) = \sin x - \cos^2 x$

$f'(x) = \cos x + 2\cos x \sin x$

$= \cos x (1 + 2\sin x)$ (B)

$\cos x = 0 \quad 1 + 2\sin x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \sin x = -1/2$

$x = \frac{7\pi}{6}, 11\pi/6$ (C)

x	$f(x)$	
0	-1	
$\pi/2$	1	← (D)
$7\pi/6$	$-\frac{1}{2} - (\frac{\sqrt{3}}{2})^2$	
$3\pi/2$	-1	
$11\pi/6$	$-\frac{1}{2} - (\frac{\sqrt{3}}{2})^2$	
2π	-1	